

lyzed. The electromagnetic fields in conductors are directly analyzed by forming sufficiently small grids compared to the skin depth, and accurate attenuation constants are obtained for the lossy structures.

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A Simple Formula for the Concentration of Charge on a Three-Dimensional Corner of a Conductor

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Abstract—A major problem in the computation of capacitance coefficients for microwave transmission and VLSI interconnection systems is caused by the singularities in the electric field at the corners and edges of conductors. For edges, a solution is given by the Duncan correction, which is based on a two-dimensional (2-D) polar expansion of the field. No such exact expansion exists for corners. Recent research by Beagles and Whiteman has yielded an asymptotic expansion for the electric field in the vicinity of a rectangular three-dimensional conductive corner, and this is used to derive a simple formula for the charge Q (in coulombs) concentrated at any such corner. The formula is $Q = 1.307 \varepsilon d(V_c - V_s)$, where ε is the dielectric permittivity (in farads per meter) of the medium surrounding the conductive corner, d is the length (in meters) of one side of a cubic region situated on the conductor adjacent to the corner, V_c is the electric potential (in volts) of the conductor, and V_s is the electric potential at a point in the medium displaced from the corner's apex along a line through the cube's diagonal and at a distance equal to that diagonal. Q is the charge on the cube's three surfaces lying along the conductor's surfaces. Such a configuration is convenient for a finite-difference computation of capacitance.

I. INTRODUCTION

This paper concerns the capacitance coefficients of three-dimensional (3-D) conductors, a matter of importance to microwave transmission networks, VLSI interconnects, power transmission systems, electric equipment, and electrical insulation technology. Much work has been done on the computation of capacitances for two-dimensional (2-D) models of interconnection lines and other conducting bodies, but much less has been accomplished for 3-D models. Extended bibliographies are given in [6], [7]. A major difficulty arises from the singularities in the electrical field at 3-D corners of a conductor. There are also field singularities along the edges of conductors, but their contributions to capacitances are readily determined by Duncan's correction [2], [5], which is based upon an exact 2-D polar expansion of the field. Since no such exact expansion exists in spherical coordinates, there is no exact 3-D analog of the Duncan correction. Instead, we have sought an approximating asymptotic expansion for the electrical field near a 3-D reentrant corner. There is a literature on this subject; see for an extended bibliography in [7]. Much of this work is of a very general nature dealing with a variety of differential equations and a variety of geometries. The paper of Beagles and Whiteman [1] is the most pertinent one for our purposes. By using the results of that work, we have derived a simple formula that takes into account the concentration of charge at any 3-D rectangular corner. This is most easily used in correcting the capacitances obtained from finite-difference computations.

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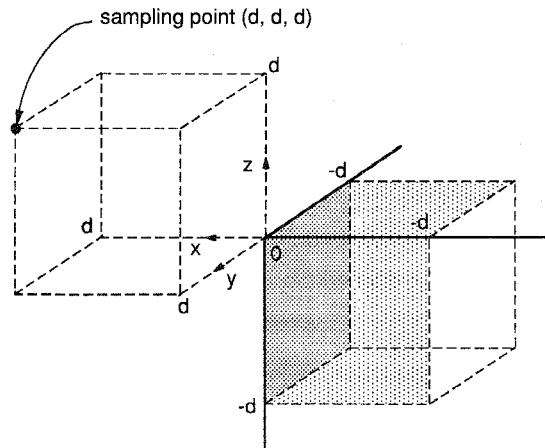


Fig. 1. A 3-D rectangular corner on a conductor. The edges of the corner lie along the negative x , y , z coordinates. The total charge given by (1) lies on the surface consisting of the three squares of length d per side shown shaded. The point in the medium at which electric potential is sampled is (d, d, d) ; thus, it lies on a line passing through the diagonal of the cube formed by the three squares and displaced from the apex O of the corner by the length of the diagonal.

II. THE FORMULA

Our objective is to determine the total charge on an incremental area adjacent to a 3-D rectangular corner of a perfect conductor. That area consists of three squares of equal size abutting the apex of the corner, as shown by the shaded areas in Fig. 1. Thus, those squares form three sides of a cube with side length of d meters. We choose rectangular coordinates (x, y, z) such that the edges of the corner lie along the negative axes. Another cube of equal size abut the apex, with the diagonals of both cubes forming a straight line. Thus, the point (d, d, d) lies in the medium surrounding the conductor and at the second cube's corner furthest away from the apex O . Let V_s be the electric potential at the point (d, d, d) and let V_c be the electric potential of the conductor, both measured in volts of course. Thus, V_s samples the electric potential in the vicinity of the corner when the conductor is held at the voltage V_c . The total charge Q in coulombs on the three squares (shown shaded in Fig. 1) is given by

$$Q = 1.307\epsilon d(V_c - V_s) \quad (1)$$

where ϵ is the dielectric permittivity of the medium in farad per meter.

This is a very simple formula to use in order to take into account the electric field singularity when the capacitance of a conductor is being determined through a finite-difference computation of the electric potential. This is especially so when the increments in the x , y , and z coordinates are all equal. Simply set d equal to that increment. If those increments are not the same, then d might be set equal to the smallest of them, and additional rectangular incremental areas may be appended to the squares to conform with the rectangular incremental areas of the finite-difference computation. Those additional areas abut edges of the conductor, and the charges on them can be obtained from Duncan's correction. In short, the procedure to use when making a finite-difference computation of capacitance is to discard the charge on the three squares as determined by that computation and to substitute in its place the charge as given by (1). (This is what one does when using the Duncan correction.)

III. DERIVATION OF THE FORMULA

The derivation is rather complicated. Space limitations prevent us from explicating all its details. Those can be found in the thesis [7]. However we can explain how (1) is derived.

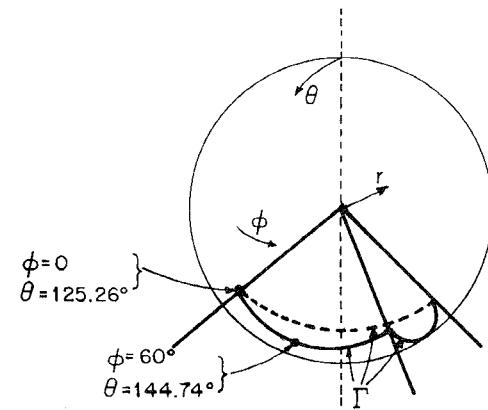


Fig. 2. A 3-D rectangular corner whose apex is at the center of a sphere. Γ is the curve of intersection between the corner and the sphere.

We work in spherical coordinates (r, θ, ϕ) with axis along the diagonal line through (d, d, d) and O in Fig. 1. This is shown in Fig. 2 with the corner rotated to make that axis vertical. Also shown is a sphere centered at the corner's apex. The sphere will intersect the corner's plane surfaces along a curve Γ . We choose ϕ to equal 0° , 120° , and 240° at the three points where the edges meet the sphere. On Γ , the minimum value of θ is 125.26° and occurs at those three edge points. Also, on Γ , the maximum value of θ is 144.74° and occurs where ϕ equals 60° , 180° , and 300° .

We wish to compute the surface charge density when the conductor is held at V_c while all other conductors—and infinity as well—are held at the voltage 0 volt.

We start with an asymptotic expansion of the electrical potential $V(r, \theta, \phi)$ in the vicinity of the corner as given in [2] and [3]

$$V(r, \theta, \phi) \approx V_c + r^\alpha \sum_{n=0}^{N-1} a_n \cos 3n\phi P_\alpha^{-3n}(\cos \theta). \quad (2)$$

Here, $P_\alpha^{-3n}(\cos \theta)$ is the associated Legendre function of the first kind of degree α and order $-3n$. The expression (2) satisfies the Laplace's equation. Furthermore, the boundary condition will be approximately but quite closely satisfied if α is chosen appropriately for the chosen N , as we shall see. Let us note at this point that $0 \leq \alpha \leq 1$, whatever be N . Indeed, if $\alpha < 0$, $V(r, \theta, \phi) \rightarrow \infty$ as $r \rightarrow 0$, which contradicts our boundary condition. Also, if $\alpha > 1$, the electrical field tends to zero as $r \rightarrow 0$; which does not account for that field's singularity at the apex. That (2) is only an asymptotic expression for $V(r, \theta, \phi)$ for small r can be seen from the fact that the right-hand side of (2) tends to infinity as $r \rightarrow \infty$.

Because of the boundary condition, we want the summation on the right-hand side of (2) to remain as close to zero as possible. Since r^α can be divided out, this need be only along the curve Γ of Fig. 2, in fact, by symmetry only along that curve from $\phi = 0^\circ$ to $\phi = 60^\circ$. Furthermore, along that arc θ is uniquely determined by ϕ , and conversely. Henceforth, we restrict Γ to that arc.

To simplify our presentation let us work with just two terms in the summation of (2), that is, let $N = 2$. We obtain two simultaneous equations for a_0 and a_1 by setting the summation equal to 0 at two points (θ_0, ϕ_0) and (θ_1, ϕ_1) on curve Γ .

In matrix form, we have

$$\begin{bmatrix} P_\alpha^0(\cos \theta_0) & \cos 3\phi_0 P_\alpha^{-3}(\cos \theta_0) \\ P_\alpha^0(\cos \theta_1) & \cos 3\phi_1 P_\alpha^{-3}(\cos \theta_1) \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} \approx 0. \quad (3)$$

Let B^α denote the 2×2 matrix in (3). In order to fulfill (3) as well as can be with nonzero a_0 and a_1 , we want $\det B^\alpha$ to remain as close

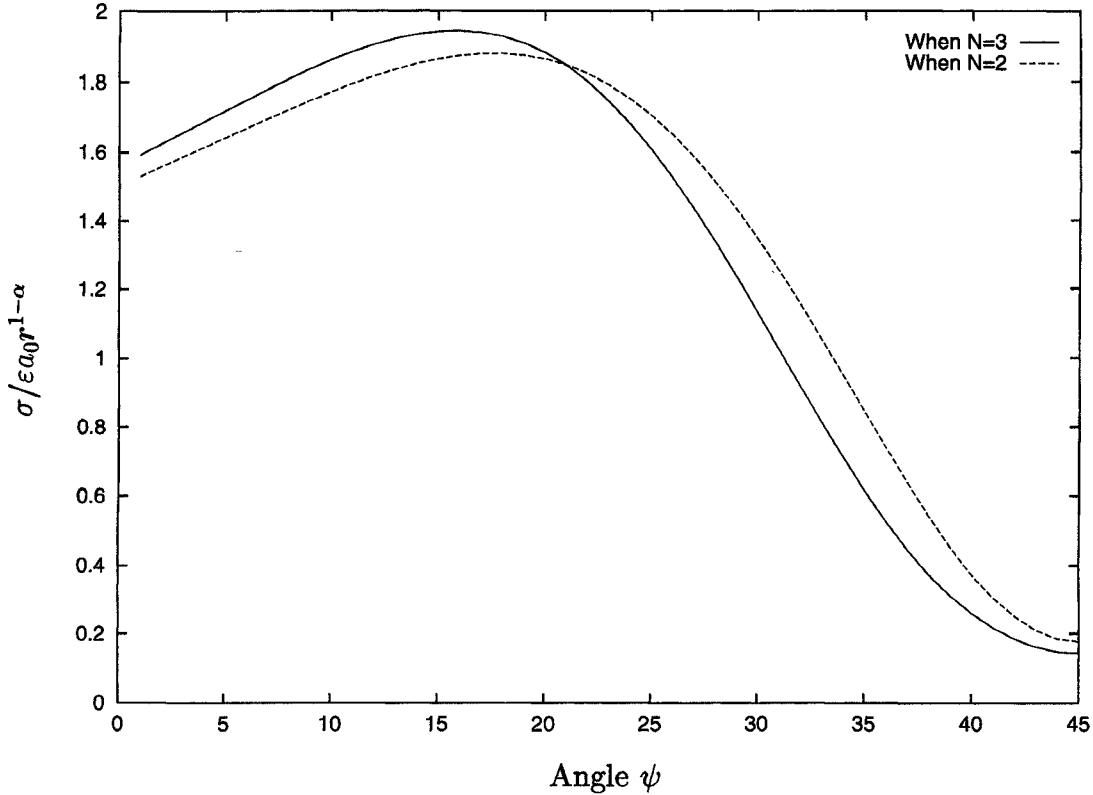
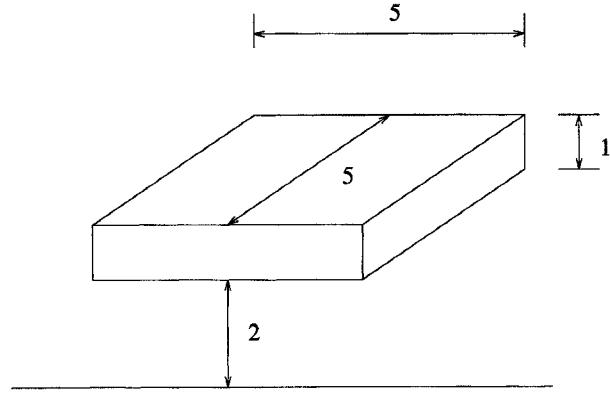
Fig. 3. Normalized charge density along the curve Γ .

Fig. 4. A rectangular conductor.

to 0 as possible for all possible choices of (θ_0, ϕ_0) and (θ_1, ϕ_1) on curve Γ . This can be accomplished by choosing α appropriately in the interval $[0, 1]$. To this end, for each α we computed the maximum value of $\det B^\alpha$ for all pairs of points on Γ . The value of α for which that maximum achieves its minimum value is $\alpha = 0.461$.

The same computation was made for three terms of the summation in (2), i.e., for $N = 3$. In this case the best value for α was 0.458. Now however B^α is 3×3 matrix, and we had to determine $\max \det B^\alpha$ for all triplets of values for (θ, ϕ) on Γ . This of course required much more computer time. Still larger values of N required more computer time than was available to us. However, our final result, namely, the numerical coefficient in (1) changed by only seven tenths of one percent between $N = 2$ and $N = 3$, being 1.316 for $N = 2$ and 1.307 for $N = 3$. So we do not expect that coefficient will change significantly for higher values of N .

As the next step, consider either one of the two approximate equalities obtained from (3). Upon factoring out a_0 , we get the

following expression for the resulting left-hand side

$$F\left(\frac{a_1}{a_0}, \psi\right) = P_{461}^0(\cos \theta) + \frac{a_1}{a_0} \cos 3\phi P_{461}^{-3}(\cos \theta). \quad (4)$$

Here, ψ is the point on Γ corresponding to the choice of (θ, ϕ) ; see (12) and (13) below. The question now is: For what value of a_1/a_0 will (4) be the smallest? To evaluate “smallest,” we used the L_1 , L_2 and L_∞ norm for the function $\psi \mapsto F(a_1/a_0, \psi)$ and obtained the same answer in each case, namely, $a_1/a_0 = -0.05$.

Altogether then, we have determined that for $N = 2$ the best expression for $V(r, \theta, \phi)$ is

$$V(r, \theta, \phi) \approx V_c + a_0 r^{461} [P_{461}^0(\cos \theta) - 0.05 \cos 3\phi P_{461}^{-3}(\cos \theta)]. \quad (5)$$

TABLE I
COMPARISON BETWEEN THE CAPACITANCES OF A RECTANGULAR CONDUCTOR (SEE FIG. 4) CALCULATED BY DIFFERENT METHODS

Method	Capacitance (F)
Singularity ignored; extension beyond edges	49.10
Singularity ignored; no extension beyond edges	34.82
Duncan correction only; no extension beyond edges	39.79
Singularity accounted for; no extension beyond edges when $N=2$	41.64
Singularity accounted for; no extension beyond edges when $N=3$	41.61
Ruehli and Brennan	42.5

TABLE II
COMPARISON BETWEEN THE RIGHT-ANGLE BEND CONDUCTOR (SEE FIG. 5) CAPACITANCES CALCULATED BY DIFFERENT METHODS

Method	Capacitance value (F)
Singularity ignored; extension beyond edges	106.06
Singularity ignored; no extension beyond edges	73.93
Duncan correction only; no extension beyond edges	79.48
Singularity accounted for; no extension beyond edges when $N=2$	81.22
Singularity accounted for; no extension beyond edges when $N=3$	81.19
Ruehli and Brennan	101.2

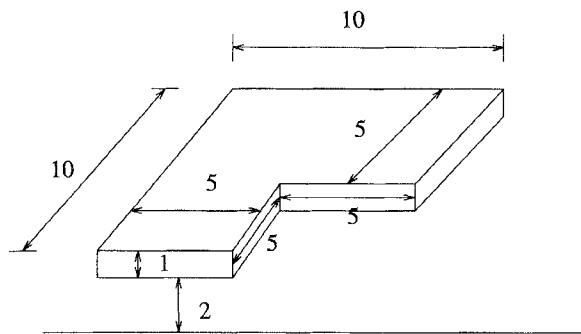


Fig. 5. A right-angle 3-D bend.

For the case of $N = 3$, we have

$$F\left(\frac{a_1}{a_0}, \frac{a_2}{a_0}, \psi\right) = P_{458}^0(\cos \theta) + \frac{a_1}{a_0} \cos 3\phi P_{458}^{-3}(\cos \theta) + \frac{a_2}{a_0} \cos 6\phi P_{458}^{-6}(\cos \theta). \quad (6)$$

Now we have to find the best pair of values a_1/a_0 and a_2/a_0 to make $\psi \mapsto F(a_1/a_0, a_2/a_0, \psi)$ the smallest. Again all three norms yielded the same result, namely, $a_1/a_0 = -0.06$ and $a_2/a_0 = -0.02$. So, for $N = 3$ the best expression for $V(r, \theta, \phi)$ is

$$V(r, \theta, \phi) \approx V_c + a_0 r^{458} [P_{458}^0(\cos \theta) - 0.06 \cos 3\phi P_{458}^{-3}(\cos \theta) - 0.02 \cos 6\phi P_{458}^{-6}(\cos \theta)]. \quad (7)$$

In both (5) and (7) the coefficient a_0 may be obtained by sampling $V(r, \theta, \phi)$ at the point (d, d, d) of Fig. 1. In spherical coordinates that sampling point is at $r = \sqrt{3}d$ and $\theta = 0$, with ϕ being indeterminate. With V_s being $V(r, \theta, \phi)$ at the sampling point, this

yields for $N = 2$:

$$a_0 = (\sqrt{3}d)^{-461} (V_s - V_c) \quad (8)$$

and for $N = 3$:

$$a_0 = (\sqrt{3}d)^{-458} (V_s - V_c). \quad (9)$$

We may now take the negative of the normal component of the gradient of $V(r, \theta, \phi)$ and multiply it by the dielectric permittivity ϵ to get the charge density $\sigma(r, \theta, \phi)$ over the squares, i.e., over the shaded regions of Fig. 1. For $N = 2$, the result is

$$\sigma = -\frac{a_0 \epsilon^{-539}}{\sin \theta} \{1.461[\cos \theta P_{461}^0(\cos \theta) - P_{1461}^0(\cos \theta)] - 0.05 \cos 3\phi [1.461 \cos \theta P_{461}^{-3}(\cos \theta) - 4.461 P_{1461}^{-3}(\cos \theta)]\} \quad (10)$$

and for $N = 3$ it is

$$\sigma = -\frac{a_0 \epsilon^{-542}}{\sin \theta} \{1.458[\cos \theta P_{458}^0(\cos \theta) - P_{1458}^0(\cos \theta)] - 0.06 \cos 3\phi [1.458 \cos \theta P_{458}^{-3}(\cos \theta) - 4.458 P_{1458}^{-3}(\cos \theta)] - 0.02 \cos 6\phi [1.458 \cos \theta P_{458}^{-6}(\cos \theta) - 7.458 P_{1458}^{-6}(\cos \theta)]\}. \quad (11)$$

An easy way to view these values is to plot them along the arc Γ from $\phi = 0^\circ$ to $\phi = 60^\circ$ for $a_0 = \epsilon = r = 1$. Moreover, points on that arc can be given in terms of a single parameter ψ relate to ϕ and θ according to

$$\cos \theta = -\sqrt{\frac{2}{3}} \sin \left(\psi + \frac{\pi}{4}\right) \quad (12)$$

$$\cos \phi = \frac{\sqrt{3} \cos \psi + \cos \theta}{\sqrt{2} \sin \theta} \quad (13)$$

where, as ϕ varies from 0 – 60° , ψ varies from 0 – 45° . The plots of σ on Γ versus ψ are given in Fig. 3. To obtain $\sigma(r, \theta, \phi)$ at arbitrary points of the squares, choose any ψ , compute θ and ϕ from (12) and (13), substitute into (11), and multiply by $a_0 \epsilon r^{\alpha-1}$. The concentration of charge at the apex of the corner is exhibited by the factor $r^{\alpha-1}$.

Observe in that Fig. 3, σ varies from a low value at $\psi = 45^\circ$, which represents a point along the diagonal of the square, to higher values as ψ decreases, i.e., as the point approaches an edge of the square, and then tapers off as ψ reaches 0, i.e., as the point reaches that edge. The initial increase is expected, but the tapering off is not because charge should concentrate more strongly near the edges of the conductor as well as at its apex. We feel that this tapering off is consequence of our truncation of (2) at low value of N . Indeed, note that, as compared to the case when $N = 2$, for $N = 3$ σ is smaller toward the center of the square, rises more sharply as the edge is approached, and then tapers off less. We expect that this improvement will continue as N is increased beyond $N = 3$. Moreover, we also feel that this tapering off does not much matter because the areas under both curves of Fig. 3 are not much different. It is the increase in this area multiplied by $r^{1-\alpha}$ which primarily determines the field singularity at the apex. Our expectation that our formula (1), which is obtained for $N = 3$, will not change by much had we chosen $N = 4$ is reinforced by the fact that the numerical coefficient in (1) changes by only seven tenths of one percent when going from $N = 2$ to $N = 3$.

The final step of our derivation is the integration of (11) over all three squares to obtain the total charge Q on all three shaded areas of Fig. 1. The result is our formula (1).

IV. TWO EXAMPLES

In order to ascertain how much corner and edge field singularities affect a typical capacitance computation and also to compare our numerical results with other results in the literature, we computed capacitances for two conductor configurations over a ground plane.

Example A: Consider the single conductor, shown in Fig. 4, of 5 m length, 5 m width, 1 m thickness, 2 m above a perfectly conducting plane, and imbedded in a medium of unit dielectric permittivity ($\epsilon = 1$). If we ignore the singularities of the electrical field at corners and edges, but allow incremental areas of the finite-difference method to extend beyond edges, the value of capacitance C is 49.10 F. If incremental areas extending beyond edges are deleted, the result is 34.82 F. If we consider edge singularities only and extended areas are deleted, the result is 39.79. On the other hand, if we consider both factors, i.e., in the most accurate case where all singularities are considered but extended areas are deleted, we get C equal to 41.64 F when $N = 2$ and 41.61 F when $N = 3$. These results can be compared to the capacitance value given by Ruehli and Brennan in [3] by interpolating in their Fig. 2 where they use the sharp edge and corner model. Their result is 42.5 F. These capacitances are listed in Table I.

Example B: Consider now a right-angle bend 2 m above a perfectly conducting plane with $\epsilon = 1$ for the medium. Its dimensions are shown in Fig. 5. If we do not consider the singularities of the electrical field at corners and edges but allow overextending incremental areas, the value of capacitance C is 106.06 F. If overextending areas are deleted, the result is 73.93 F. If we consider edge singularities only and extended areas are deleted, the result is 79.48. On the other hand, if we correct both factors, we get C is 81.22 F for $N = 2$ and 81.19

F for $N = 3$. These results can be compared to the capacitance value given by Ruehli and Brennan in [4] by interpolating in their Fig. 9; their value is 101.2 F. These capacitances are listed in Table II.

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Computation of Equivalent Circuits of CPW Discontinuities Using Quasi-Static Spectral Domain Method

D. Mirshekar-Syahkal

Abstract—An efficient and simple computer technique based on the quasistatic approximation for the determination of the component values of the equivalent circuits of a broad-class of coplanar waveguide (CPW) discontinuities is introduced. The technique does not depend on the extraction of the component values from the scattering parameters. It uses the spectral domain formulation in conjunction with the method of moments. The concepts behind the method are illustrated using a complex example, the CPW T-junction. A few T-junctions are treated with the technique. Both the measurements and the full-wave electromagnetic simulation support the accuracy of the results.

I. INTRODUCTION

For some of its advantages, the coplanar waveguide (CPW) is preferred to the microstrip line in developing monolithic microwave integrated circuits (MMIC's) on GaAs and InP. This transmission line has also found extensive applications at mm-wave and terahertz frequencies [1]. However, due to lack of powerful software, design of CPW circuits is difficult and time consuming. A comprehensive software should contain the equivalent circuits of various CPW discontinuities including bends, open and short terminations, steps, etc.

Recently, Naghed and Wolf [2], Naghed *et al.* [3], and Abdo-Tuko *et al.* [4] used the three-dimensional finite difference method to char-

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